Government of Western Australia
Curriculum Council

WACE
Western Australian Certificate of Education

MATHEMATICS

2C/2D

Calculator-free

WACE Examination 2010

Final Marking Key

This 'stand alone' version of the WACE Examination 2010 Final Marking Key is provided on an interim basis.

The Standards Guide for this examination will include the examination questions, marking key, question statistics and annotated candidate responses. When the Standards Guide is published, this document will be removed from the website.

Question 1 **(6 marks)**

Solve each of the following equations.

(a) $3x^2 - 15x = 0$ (3 marks)

Solution
$3x^2 - 15x = 0$ $3x(x - 5) = 0$ $x = 0$ or $x = 5$
Specific Behaviours
✓ factorises expression ✓✓ solves for both values of x

(b) $(x + 1)(x - 2) = 4$ (3 marks)

Solution
$x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$ $x = -2$ or $x = 3$
Specific Behaviours
✓ follows conventions when rearranging algebraic terms ✓✓ solves for both values of x

Question 2 **(3 marks)**

(a) Simplify

$$\frac{2^5 \times 2^3}{2^4}$$

(1 mark)

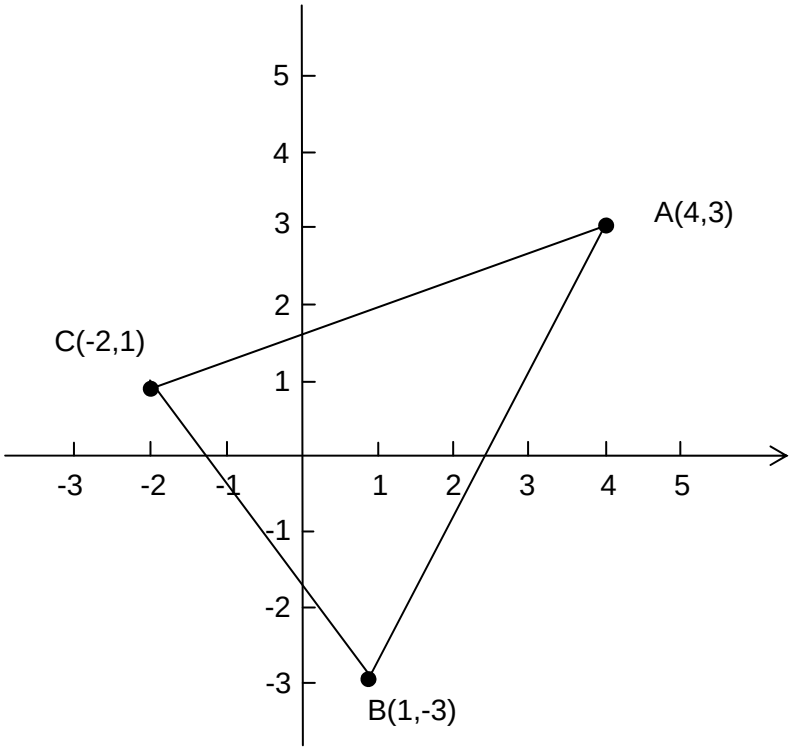
Solution
$\frac{2^5 \times 2^3}{2^4}$ $= 2^{(5+3-4)}$ $= 2^4 = 16$
Specific Behaviours
✓ simplifies by adding and subtracting powers

(b) Estimate the solution to the equation $\frac{14}{2^x} = 1$ to the nearest whole number. Justify your answer. (2 marks)

Solution
$\frac{14}{2^x} = 1$ $14 = 2^x$ Testing: $2^3 = 8$ $\quad \frac{14}{8} = \frac{7}{4} = 1.75$ $2^4 = 16$ $\quad \frac{14}{16} = \frac{7}{8}$ $\therefore x = 4$
Specific Behaviours
✓ simplifies equation ✓ tests values to decide nearest estimate (1 for correct value; 1 for justification)

Question 3 **(7 Marks)**

The diagram below shows the position of three mine shafts A(4, 3), B(1, –3) and C(–2, 1), relative to the processing plant that is located at the origin (0, 0). All units are in kilometres.



(a) Determine the gradient of the line passing through AB. (1 mark)

Solution
2
Specific Behaviours
✓ uses co-ordinates of A and B to determine gradient.

(b) What is the gradient of the line perpendicular to the side AB? (1 mark)

Solution
$-\frac{1}{2}$
Specific Behaviours
✓ recognises the relationship between gradients of perpendicular lines.

(c) Determine the equation of the line that is perpendicular to the side AB and passes through the point C. (2 marks)

Solution		
$(y-1)=-\frac{1}{2}(x+2)$ $y=-\frac{1}{2}x-1+1$ $y=-\frac{1}{2}x$	**or**	$y=mx+c$ $1=-\frac{1}{2}(-2)+c$ $1=1+c$ $c=0$ $\therefore y=-\frac{1}{2}x$
Specific Behaviours		
✓ substitutes gradient and co-ordinates of C into equation. ✓ states equation of line in $y = mx + c$ form.		

(d) Determine the distance between the mine shaft at A and the processing plant. (1 mark)

Solution
$\sqrt{(4-0)^2+(3-0)^2}=\sqrt{25}=5\text{km}$
Specific Behaviours
✓ determines correct distance

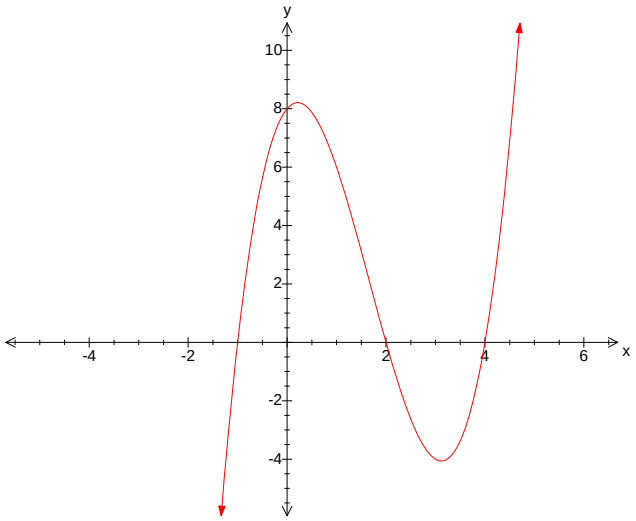
(e) Mary needs to drive from B to C, while John needs to return to the processing plant from A. Assuming that they both start to travel at the same time and at the same speed, determine who will be the first to arrive at their destination. Justify your answer. (2 marks)

Solution
$\sqrt{(1+2)^2+(-3-1)^2}=\sqrt{25}=5\text{km}$ Therefore, Mary and John will take exactly the same time to reach their destination.
Specific Behaviours
✓ determine correct distance for John ✓ makes correct conclusion based on distances

Question 4 **(5 marks)**

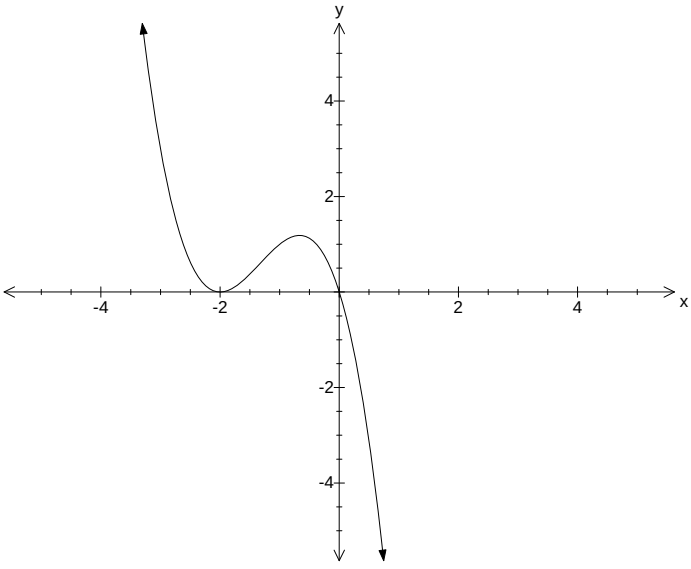
Draw a neat sketch of each function.

(a) $y = (x + 1)(x - 2)(x - 4)$ (3 marks)



Solution
Graph as above
Specific Behaviours
✓ displays the correct orientation
✓ identifies correct y-intercept
✓ identifies correct x-intercepts

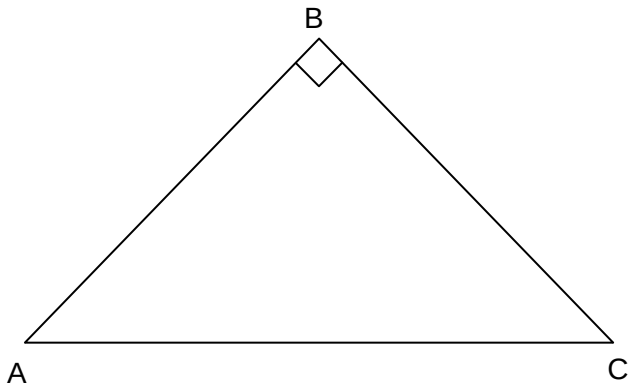
(b) $y = -x(x+2)^2$ (2 marks)



Solution
as above
Specific Behaviours
✓ recognises minimum at (–2, 0)
✓ passes through (0, 0)

Question 5 **(4 marks)**

Consider the following triangle.



In the triangle above, $\cos \angle BAC = 0.8$.

(a) If the length of AC is 100 cm, calculate the length of AB. (2 marks)

Solution
$\cos \angle BAC = \frac{AB}{AC} = .80$ $AB = .80 \times AC$ $= 80\text{cm}$
Specific Behaviours
✓ recognises the cosine ratio as 8 to 10 ✓ calculates the length of AB

(b) Evaluate $\tan \angle ACB$. (2 marks)

Solution
$\tan \angle ACB = \frac{AB}{BC} = \frac{80}{BC}$ $= \frac{80}{60}$ $= \frac{4}{3} \quad \left(\text{or } 1.3\dot{3}\right)$
Specific Behaviours
✓ recognises the tangent ratio as 80 to 60 ✓ correctly determines $\tan \angle ACB$

Question 6 **(12 marks)**

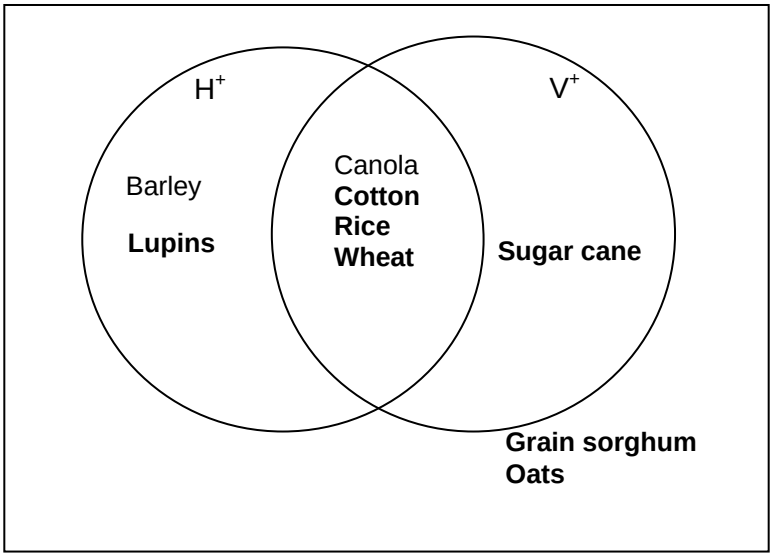
Australian agriculture is important for food production and export earnings. The table compares the harvest (000 t) and value ($million) of some crops in Australia for the years ending June 2008 and June 2009.

Australian agriculture, years ending June 2008 and June 2009

		Harvest (000t)		Value ($million)	
	Crop	**2008**	**2009**	**2008**	**2009**
Harvest up in 2009	Barley	7 160	7 669	2 244	1 767
	Canola	1 214	1 861	659	1 026
	Cotton	119	303	227	623
	Lupins	662	716	222	202
	Rice	18	63	7.3	35.5
	Wheat	13 569	20 939	5 292	5 894
Harvest down in 2009	Grain sorghum	3790	2671	977	550
	Oats	1502	1205	423	255
	Sugar cane	32 621	30284	861	983

Let H^+ denote the set of crops for which the harvest was greater in the year ending June 2009 than in the year ending June 2008 and V^+ denote the set of crops whose value was greater in the year ending June 2009 than in the year ending June 2008.

(a) Complete the Venn diagram for sets H^+ and V^+. (4 marks)



Solution
as above
Specific Behaviours
✓ correct entries in the intersection ✓ correct entry – lupins ✓ correct entry – sugar cane

✓	correct entries – grain sorghum, oats.

(b) Explain the real-life meaning of $n(H^+ \cap V^+)$ and find its value. (2 marks)

Solution
The number of crops for which the harvest and value increased in the year ending 2009 compared to the year 2008. $n(H^+ \cap V^+) = 4$
Specific Behaviours
✓ identifies intersection of sets ✓ correct value.

(c) Evaluate

(i) $P(H^+ \cup V^+)$ (1 mark)

Solution
$= \frac{7}{9}$
Specific Behaviours
✓ calculates probability based on (a)

(ii) $P(\overline{H^+ \cup V^+})$ (1 mark)

Solution
$= \frac{2}{9}$
Specific Behaviours
✓ calculates probability based on (a)

(iii) $P(\overline{H}^+ \cup \overline{V}^+)$ (2 marks)

Solution
$= \frac{5}{9}$
Specific Behaviours
✓✓ calculates probability based on (a)

(d) (i) Express the following question using probability notation:

'Given that the value of a crop for the year ending June 2009 was greater than the value for the year ending June 2008, what is the probability that the harvest (tonnes) increased?' (1 mark)

Solution	
	$P(H^+ \mid V^+)$
Specific Behaviours	
✓	correct notation

(ii) Determine the answer to the question in (i). (1 mark)

Solution	
	$\frac{4}{5}$
Specific Behaviours	
✓	correctly determines probability

Question 7 **(3 marks)**

The following is a list of all prime numbers less than 20.

2, 3, 5, 7, 11, 13, 17, 19

Kate looked at this list and came up with the following conjecture:

'Every integer greater than three can be written as the sum of two prime numbers.'

(a) Show calculations for four different integers to test whether this conjecture might be true. (2 marks)

Solution
4 = 2 + 2 5 = 2 + 3 6 = 3 + 3 7 = 2 + 5 8 = 3 + 5 9 = 2 + 7 10 = 5 + 5 11 = 5 + 6 *
Specific Behaviours
✓ tests conjecture with two examples ✓ tests conjecture with two examples

(b) Give your conclusion to the conjecture, based on your results in (a). (1 mark)

Solution		
False. Not true for 11	**or**	It appears to be true because all values in (a) can be written as the sum of two prime numbers, however, we have not tested all possible values and cannot conclusively say it is true.
Specific Behaviours		
✓ makes valid statement based on (a)		

ACKNOWLEDGEMENT

Section One

Question 6 Data source: Australian Bureau of Statistics. (n.d.). Retrieved March, 2010, from www.abs.gov.au.